UAM-A. DEPARTAMENTO DE CIENCIAS BÁSICAS

EXAMEN GLOBAL MATUTINO DE CÁLCULO INTEGRAL

Trimestre 22P

El examen global consta de los 10 problemas marcados con ♣. No se permite el uso de formularios. **Si se presenta sólo una parte, se debe resolver TODOS los ejercicios de esa parte**. Todos los resultados deben mostrar el procedimiento correspondiente.

PRIMERA PARTE

1. ♣ **(10 puntos)** Obtener el valor de $G'(1)$, si

$$G(t) = e^{t^2} \cdot \int_{\pi/4}^{\arctan(t)} e^{-x^2}\, dx$$

2. Calcular la integral:

$$\int \frac{(1+\sin(x))^2}{\cos^2(x)}\, dx$$

Calcular las integrales:

3. $\displaystyle\int_1^3 \frac{|4-2x|}{x}\, dx$ **4.** ♣ **(10 puntos)** $\displaystyle\int_1^{25} \frac{\sqrt{4+\sqrt{x}}}{\sqrt{x}}\, dx$ **5.** ♣ **(10 puntos)** $\displaystyle\int_0^1 x^2 e^{-2x}\, dx$

SEGUNDA PARTE

Resolver

6. $\displaystyle\int \cos^{1/4}(x)\sin^3(x)\, dx$ **7.** ♣ **(15 puntos)** $\displaystyle\int \frac{\sqrt{9-x^2}}{x^4}\, dx$ **8.** ♣ **(15 puntos)** $\displaystyle\int \frac{x+2}{x^4+4x^2}\, dx$

9. ♣ **(10 puntos)** Calcular el valor de la siguiente integral impropia: $\displaystyle\int_0^1 \frac{\ln(x)}{x^2}\, dx$

TERCERA PARTE

10. ♣ **(10 puntos)** Calcular el área de la región limitada por las gráficas de $8y^2 - x - 3 = 0$, $2y^2 - x + 3 = 0$.

11. ♣ **(10 puntos)** Calcular el volumen del sólido de revolución obtenido al rotar alrededor del eje X la región limitada por las gráfica de $f(x) = 1 + \sin(x)$, $g(x) = \sin(x)$ en el intervalo $x = 0$, $x = \pi$.

12. ♣ **(10 puntos)** Calcular la longitud de la gráfica de la función $y = 1 - \ln(\cos(x))$, en el intervalo $[0, \frac{\pi}{4}]$.

Solución al examen global matutino de Cálculo Integral

1. Si $G(t)=e^{t^2}\cdot\int_{\frac{\pi}{4}}^{\text{ArcTan}(t)} e^{-x^2}dx$, entonces

$$G'(t)=e^{t^2}\cdot e^{-(\text{ArcTan}(t))^2}\cdot\frac{1}{1+t^2}+2te^{t^2}\int_{\frac{\pi}{4}}^{\text{ArcTan}(t)} e^{-x^2}dx,$$

$$G'(1)=e^{-\frac{\pi^2}{16}}\cdot\frac{1}{2}$$

2. $$\int\frac{(1+\operatorname{sen}x)^2}{\cos^2x}dx=\int\frac{1+2\operatorname{sen}x+\operatorname{sen}^2x}{\cos^2x}dx=\int\sec^2x\,dx+2\int\frac{\operatorname{sen}x}{\cos^2x}dx+\int\tan^2x\,dx$$

$$=\tan x+\frac{2}{\cos x}+\tan x-x+C$$

3. $$\int_1^3\frac{|4-2x|}{x}dx=\int_1^2\frac{4-2x}{x}dx+\int_2^3\frac{2x-4}{x}dx=\left(4\ln|x|-2x\right)\Big|_1^2+\left(2x-4\ln|x|\right)\Big|_2^3$$

$$=(4\ln2-4)-(-2)+(6-4\ln3)-(4-4\ln2)$$

$$=8\ln2-4\ln3$$

4. $$\int_1^{25}\frac{\sqrt{4+\sqrt{x}}}{\sqrt{x}}dx=\int_1^5\sqrt{4+y}\,dy=\frac{2}{3}(4+y)^{3/2}\Big|_1^5=\frac{2}{3}(9)^{3/2}-\frac{2}{3}(5)^{3/2}$$

$(y=\sqrt{x};\ dx=2y\,dy)$

5. $$\int_0^1 x^3 e^{-2x}\,dx = -\frac{1}{2}x^3e^{-2x} + \frac{3}{2}\int x^2 e^{-2x}\,dx$$

$\begin{pmatrix} u_1 = x^3 & ; du_1 = 3x^2 \\ dv_1 = e^{-2x}dx & ; v_1 = -\frac{1}{2}e^{-2x} \end{pmatrix}$ $\begin{pmatrix} u_2 = x^2 & ; du_2 = 2x\,dx \\ dv_2 = e^{-2x}dx & ; v_2 = -\frac{1}{2}e^{-2x} \end{pmatrix}$ $\begin{pmatrix} u_3 = x\,dx & ; du_3 \\ dv_3 = e^{-2x} & ; v_3 = -\frac{1}{2}e^{-2x} \end{pmatrix}$

$$= -\frac{1}{2}x^3e^{-2x} + \frac{3}{2}\left[-\frac{1}{2}x^2e^{-2x} + \int x e^{-2x}dx\right] = -\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} + \frac{3}{2}\int x e^{-2x}dx$$

$$= -\frac{1}{2}x^3e^{-2x} - 3x^2e^{-2x} - \frac{3}{4}xe^{-2x} - \frac{3}{8}e^{-2x}\Big|_0^1$$

$$= \left(-\frac{1}{2}e^{-2} - 3e^{-2} - \frac{3}{4}e^{-2} - \frac{3}{8}e^{-2}\right) - \left(-\frac{1}{2} - 3 - \frac{3}{4} - \frac{3}{8}\right)$$

$$= -\frac{37}{8}e^{-2} + \frac{37}{8} = \frac{37}{8}\left(1 - e^{-2}\right)$$

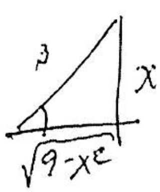
6. $$\int \cos^{1/4}x\,\mathrm{sen}^3x\,dx = \int \cos^{1/4}x\,(1-\cos^2x)\,\mathrm{sen}\,x\,dx = -\int\left(u^{1/4} - u^{9/4}\right)du$$

$\begin{pmatrix} u = \cos x \\ du = -\mathrm{sen}\,x\,dx \end{pmatrix}$ $$= -\frac{4}{5}u^{5/4} - \frac{04}{13}u^{13/4} + C = -\frac{4}{5}\cos^{5/4}x - \frac{4}{13}\cos^{13/4}x + C$$

7. $$\int \frac{\sqrt{9-x^2}}{x^4}\,dx = \int \frac{3\cdot\cos\theta}{3^4\,\mathrm{sen}^4\theta}\cdot 3\cos\theta\,d\theta = \frac{1}{9}\int\frac{\cos^2\theta}{\mathrm{sen}^4\theta}\,d\theta$$

$\begin{pmatrix} x = 3\,\mathrm{sen}\,\theta \\ dx = 3\cos\theta\,d\theta \end{pmatrix}$ $$= \frac{1}{9}\int \cot^2\theta\,\csc^2\theta\,d\theta = -\frac{1}{9}\frac{1}{3}\cot^3\theta + C$$

$\begin{pmatrix} w = \cot\theta \\ dw = -\csc^2\theta\,d\theta \end{pmatrix}$ $$= -\frac{1}{27}\left(\frac{\sqrt{9-x^2}}{x}\right)^3 + C$$



8. $\displaystyle\int \frac{x+2}{x^4+4x^2}\,dx = \int \frac{x+2}{x^2(x^2+4)}\,dx$

$$\frac{x+2}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} = \frac{Ax(x^2+4)+B(x^2+4)+(Cx+D)(x)}{x^2(x^2+4)}$$

$$A=0;\ B=\tfrac{1}{2},\ C=-\tfrac{1}{2},\ D=1$$

$$\int \frac{x+2}{x^2(x^2+4)}\,dx = \int \frac{\frac{1}{2}}{x^2}\,dx + \int \frac{\frac{1}{2}x+1}{x^2+4}\,dx = -\frac{1}{2x} + \frac{1}{4}\ln(x^2+4) + \frac{1}{2}\operatorname{ArcTan}\frac{x}{2}+c$$

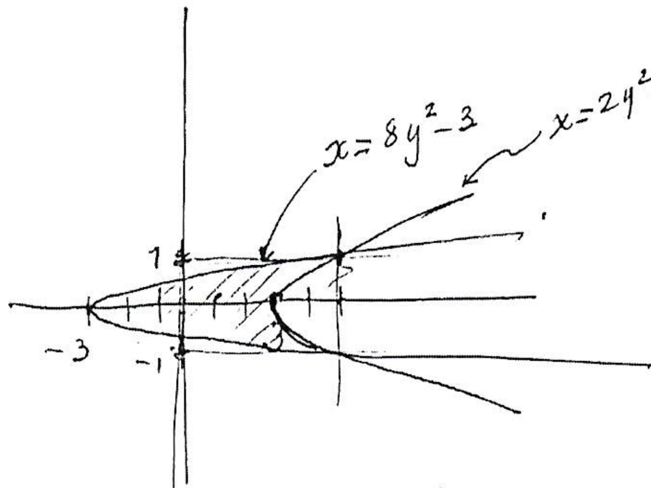
9. $\displaystyle\int_0^1 \frac{\ln x}{x^2}\,dx = \lim_{a\to 0^+}\int_a^1 \frac{\ln x}{x^2}\,dx$

$$\int_a^1 \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x}\Big|_a^1 + \int_a^1 \frac{1}{x^2}\,dx = \frac{\ln a}{a} - \frac{1}{x}\Big|_a^1$$

$$= \frac{\ln a}{a} - 1 + \frac{1}{a} = -1 + \frac{1+\ln a}{a}$$

$$\lim_{a\to 0^+}\int_a^1 \frac{\ln x}{x^2}\,dx = -1 + \lim_{a\to 0^+}\frac{1+\ln a}{a} = -1 + \lim_{a\to 0^+}(1+\ln a)\left(\frac{1}{a}\right) = \cancel{+\infty} = -\infty$$

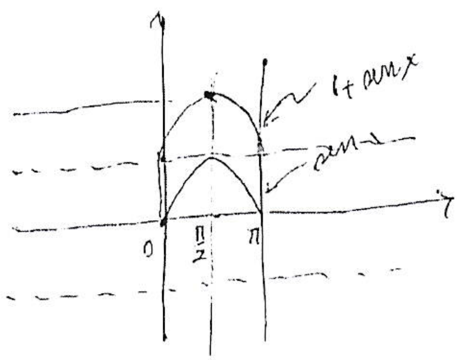
10.



$$A = \int_{-1}^{1}\left[(2y^2+3)-(8y^2-3)\right]dy = \int_{-1}^{1}(-6y^2+6)\,dy = -\frac{6y^3}{3}+6y\Big|_{-1}^{1}$$

$$= 2y^3+6y\Big|_{-1}^{1} = (2+6)+(-2+6) = 4+12 = 16$$

11.



$$V = \pi\int_0^{\pi}\left[(1+\operatorname{sen} x)^2-\operatorname{sen}^2 x\right] = \pi\int_0^{\pi}(1+2\operatorname{sen} x)dx$$

$$V = \pi\,(x-2\cos x)\Big|_0^{\pi} = \pi\left[(\pi-2\cos\pi)-(0-2\cos 0\right.$$

$$V = \pi\left[\pi+2+2\right] = \pi(\pi+4)$$

12.

$$L = \int_0^{\frac{\pi}{4}}\sqrt{1+\left[\frac{d}{dx}(\ln|\cos x|)\right]^2}\,dx = \int_0^{\frac{\pi}{4}}\sqrt{1+\tan^2 x}\,dx = \int_0^{\frac{\pi}{4}}\sec x\,dx$$

$$= \ln|\sec x+\tan x|\,\Big|_0^{\frac{\pi}{4}} = \ln\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right| = \ln\left|\frac{2}{\sqrt{2}}+1\right|$$